Assignment 7 – SOLUTION

METHOD 1

One way to go about this is to set up some (fancy) algebraic expressions for shear and bending moment and solve for the `w' values directly, namely,

1. Let $M = w_{sw}L^2/8 + w_{DL}{}^2/8 + w_{LL}L^2/8 = M_{cr} = f_r S$ ...

   where $w_{DL} = 1/3\ w_a$ and $w_{LL} = 2/3\ w_a$ ... call it w_{cr}.

   and solve for w_a.

2. Let $V = w_{sw}L/2 + w_{DL}L/2 + w_{LL}L/2 = V_c = 2bd\sqrt{f_c'}$ ... solve for w_a ($w_{a,v}$)

   ... being careful to use the correct `L' values.

3. Let $M = w_{sw}L^2/8 + w_{DL}{}^2/8 + w_{LL}L^2/8 = M_{ult}$ ... = solve for w_a ($w_{a,\ ult}$).

4. Let $V_u = 1.2\ w_{sw}L/2 + 1.2\ w_{DL}L/2 + 1.6\ w_{LL}L/2 = \phi\ V_c = \phi\ 2bd\sqrt{f_c'}$ ... (bringing in the factors of safety) then solve for w_a ($w_{a,\ safe,\ v}$).

   and

5. Let $M_u = 1.2\ w_{sw}L^2/8 + 1.2\ w_{DL}{}^2/8 + 1.6\ w_{LL}L^2/8 = \phi\ M_{ult}$ ... = solve for w_a ($w_{a,\ safe,\ ult}$).

And then of course choose which `safe' load governs.

METHOD 2

Solve for M_{cr}, V_c, M_{ult}, ϕ Vc, and $\phi\ M_{ult}$ … (which we already did in class) …

Then set up equations for …

$M = w_{sw}L^2/8 + w_{DL}{}^2/8 + w_{LL}L^2/8$ ... $w_{DL} = 1/3\ w_a$ ... blah, blah ...

and

$V = w_{sw}L/2 + w_{DL}L/2 + w_{LL}L/2$

And with the factors of safety ...

$V_u = 1.2\ w_{sw}L/2 + 1.2\ w_{DL}L/2 + 1.6\ w_{LL}L/2$

and

$M_u = 1.2\ w_{sw}L^2/8 + 1.2\ w_{DL}{}^2/8 + 1.6\ w_{LL}L^2/8.$

And then just start dumping in values of w_a, say, starting with 0, and increasing by, say, 50 or 100 plf at a time, until the numbers that crank out hit our various strength values ...

LET'S DO METHOD 2 ...

First let's calculate M_{cr}, V_c, and M_{ult} ...

Actually, we already did all these in class ...

But, again, for practice! ...

$M_{cr} = f_r S$

f_r = 7.5 sqrt (f_c') = 7.5 sqrt (3000) = 411 psi.

$S = bh^2/6 = 5(9)^2 / 6 = 67.5$ in.^{3}

M_{cr} = 411 psi (67.5 in.^{3}) = 27,740 lb-in. = 2312 lb-ft.

V_c = 2 b d sqrt f_c' = 2 (5)(7) sqrt (3000) = 3834 lb.

$M_{ult} = A_s f_y d^*$

$\rho = A_s / (bd) = 0.31/(5x7) = 0.00886$

[...] = [1 - .59 (0.00886)(60,000)/3000] = 0.895 ... or just read it off that chart I posted ... for 0.0089 ... gives 0.895 ... same.

So, d* = 0.895(7) = 6.265 in.

M_{ult} = (0.31)(60,000)(6.265) = 116,530 lb-in. = 9711 lb-ft.

Now for the loads ...

Shear ...

V_a = shear load (near support) due to self weight applied loads

$V_a = V_{\text{due to self weight}} + V_{\text{due to applied dead load}} + V_{\text{due to applied live load}}$

$V_{\text{self weigtht}} = w_{\text{self weight}} L_{\text{for self weight}} / 2$

The $L_{\text{for self weight}}$ we found in class ... namely L – 2 x ½ of h = 6.0 ft – 2 x ½ of 9/12 ft = 5.25 ft

And w = 150 pcf (5/12 ft)(9/12 ft) = 46.9 plf.

So $V_{\text{self weigtht}}$ = 46.9 plf (5.25 ft) /2 = 123 lb.

$V_{\text{due to applied dead load}} = 1/3\ w_a\ L_{\text{for applied load to top of beam}} / 2$ ...

Where $L_{\text{for applied load to top of beam}}$ we talked about in class being L – 2 d ... so 6.0 ft – 2 (7/12 ft) = 4.83 ft.

So, $V_{\text{due to applied dead load}} = 1/3\ w_a$ 4.83 ft / 2 ...

$V_{\text{due to applied live load}}$ = similarly 2/3 w_a 4.83 ft / 2.

You'll see why I keep DL and LL separate after a while.

Now for moments.

$M_a = w_{\text{self weight}} L^2 / 8 + 1/3\ w_a L^2 / 8 + 2/3\ w_a L^2 / 8$ ... where the L is the same for all ... 6.0 ft.

Now I'll dump this into a spreadsheet and calc the V_a and M_a for various load levels of w_a.

Also, in the spreadsheet, I'll multiply the D loads by 1.2 and L by 1.6 to come up with the `u' loads.

And I'll play around with w_a until these factored loads hit my factored strengths ...

ϕV_c = 0.75 (3834 lb) = 2876 lb, and

ϕM_{ult} = 0.9 (9711 lb) = 8740 lb-ft.

(... the Strength reduction factors from, say, Ambrose p. 377)

Assignment 7 Calculation ...

no.	w self wt. (plf)	applied wa (plf)	Va = (lb)	Ma = (lb-ft)	Factored Loads Va,u = (lb)	Ma, u = (lb-ft)	
1	47	0	123	211	148	253	
2	47	100	365	661	502	913	
3	47	200	606	1,111	856	1,573	
4	47	300	848	1,561	1,210	2,233	
5	47	400	1,089	2,011	1,564	2,893	
6	47	500	1,331	2,461	1,919	3,553	... near M_{cr} (2312)
7	47	600	1,572	2,911	2,273	4,213	
8	47	700	1,814	3,361	2,627	4,873	
9	47	800	2,055	3,811	2,981	5,533	... near ϕV_c (2876)
10	47	900	2,297	4,261	3,335	6,193	
11	47	1,000	2,538	4,711	3,690	6,853	
12	47	1,100	2,780	5,161	4,044	7,513	
13	47	1,200	3,021	5,611	4,398	8,173	
14	47	1,300	3,263	6,061	4,752	8,833	... near ϕM_{ult} (8740)
15	47	1,400	3,504	6,511	5,106	9,493	
16	47	1,500	3,746	6,961	5,461	10,153	
17	47	1,600	3,987	7,411	5,815	10,813	... near V_c (3834 lb)
18	47	1,700	4,229	7,861	6,169	11,473	
19	47	1,800	4,470	8,311	6,523	12,133	
20	47	1,900	4,712	8,761	6,877	12,793	
21	47	2,000	4,953	9,211	7,232	13,453	
22	47	2,100	5,195	9,661	7,586	14,113	... near M_{ult} (9711)

Getting closer ... (trial and error)

		wa	Va	Ma	Vu	Mu	
crack	47	467	1,251	2,313	1,802	3,335	... hit M_{cr} (2312)
safe shear	47	770	1,983	3,676	2,875	5,335	... hit ϕV_c (2876)
safe ult	47	1,285	3,226	5,994	4,699	8,734	... hit ϕM_{ult} (8740)
shear	47	1,535	3,830	7,119	5,585	10,384	... hit V_c (3834 lb)
ult	47	2,110	5,219	9,706	7,621	14,179	... hit M_{ult} (9711)

So,

w cr = 467 plf

w v,safe = 770 plf

w ult, safe = 1285 plf

w v = 1535 plf

w_{ult} = 2110 plf

So,

The load w_{cr} that will crack the beam is ... about 470 plf (about 500 plf)
The load that will `break' (perfect world) the beam is w_{break} = 1535 plf ... and it will *shear* (say 1500 plf)
The safe load $w_{a, safe}$ is 770 plf (about 800 plf ... and based on shear).

I kind of like it this way because it shows what will happen first ... kind of like in a real loading ...